

Two-Dimensional Dilute Ising Models: Critical Behavior near Defect Lines

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We consider two-dimensional Ising models with randomly distributed ferromagnetic bonds and study the local critical behavior at defect lines by extensive Monte Carlo simulations. Both for *ladder*- and *chain*-type defects, nonuniversal critical behavior is observed: the critical exponent of the defect magnetization is found to be a continuous function of the strength of the defect coupling. Analyzing corresponding stability conditions, we obtain new evidence that the critical exponent ν of the bulk correlation length of the random Ising model does not depend on dilution, i.e., $\nu = 1$.

KEY WORDS: Random Ising model; defect lines; Monte Carlo simulations.

1. INTRODUCTION

The presence of quenched randomness may drastically change the critical properties of magnetic systems. For disorder which is coupled to the energy density, i.e., in particular for random bond and random site dilution, the relevance-irrelevance of the perturbation at a second-order phase transition point is given by the well known Harris criterion.⁽¹⁾ If the specific heat exponent of the pure system is positive, $\alpha > 0$, a new random fixed point is expected to control the critical properties of the dilute model. The marginal situation in the Harris criterion, $\alpha = 0$, is represented by the two-dimensional (2d) Ising model, in which case detailed studies, both (field-) theoretical^(2, 3) and numerical⁽⁴⁻⁶⁾ have been performed to clarify the critical properties of the dilute model. By now, according to general view, the dilution is considered as a marginally irrelevant perturbation, thus the critical

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singularities in the dilute Ising model are characterized by the power laws of the perfect model modified by logarithmic corrections. There is, however, another view which interprets numerical data as giving evidence for dilution dependent critical exponents.⁽⁷⁾

In this paper we try to decide between the conflicting views in an indirect way by studying the local critical behavior at a defect line⁽⁸⁾ in the dilute model. A defect line, which could be located at grain boundaries in real systems, represents a marginal perturbation in the 2d perfect Ising model. According to Bariev's exact solution⁽⁹⁾ the critical exponent β_d , defined via the temperature dependence of the defect or local magnetization m_d ,

$$m_d \sim t^{\beta_d}, \quad t = (T_c - T)/T_c \rightarrow 0^+ \quad (1)$$

is a continuous function of the strength of the defect coupling J_d . For a chain defect, see Fig. 1b, one gets

$$\beta_d = \frac{2}{\pi^2} \arctan^2 \kappa_c, \quad \kappa_c = \exp[-2(J_d - J)/T_c] \quad (2)$$

whereas for a ladder defect, see Fig. 1a, β_d is given by

$$\beta_d = \frac{2}{\pi^2} \arctan^2 \kappa_l, \quad \kappa_l = \frac{\tanh(J/T_c)}{\tanh(J_d/T_c)} \quad (3)$$

where J is the coupling in the isotropic Ising model. We note that the above formulae could be generalized to non-isotropic models, as well.⁽⁹⁾ As

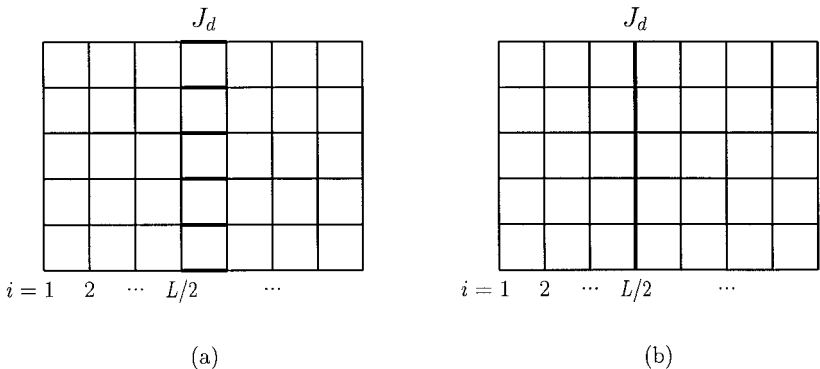


Fig. 1. Ladder (a) and chain (b) type defects.

shown recently by Pleimling and Selke,⁽¹⁰⁾ the edge magnetization of three-dimensional Ising magnets at the surface transition has a similar non-universal critical behavior, which, indeed, can be related to the local critical behavior at a defect line in the two-dimensional Ising model.

The exact results on the local critical behavior of the 2d Ising model in Eqs. (2) and (3) are in complete agreement with a stability analysis of the fixed point of the homogeneous system in the presence of a defect line.^(11, 8) Under a small perturbation this fixed point is unstable, if the critical exponent of the bulk correlation length, ν , is $\nu \leq 1$. Furthermore, a ladder defect with small local couplings behaves like two weakly coupled surfaces, and ordinary surface critical behavior will result, provided the corresponding surface fixed point remains stable against a weak coupling between the surfaces.^(11, 8) This stability condition can be expressed in terms of the surface susceptibility exponent of the homogeneous model as $\gamma_{1,1} < 0$.⁽⁸⁾ Applying hyperscaling, one obtains for $d=2$, $\gamma_{1,1} = \nu - 2\beta_1$, where β_1 is the critical exponent of the surface magnetization). As one may easily check the corresponding two marginality conditions, for ladder defects,

$$\nu = 1 \quad \text{and} \quad \gamma_{1,1} = 0 \quad (4)$$

are both satisfied for the 2d Ising model; the marginality is manifested by the defect coupling dependent critical exponent in Eq. (3).

In the following, we are going to utilize the above observations and study the local critical behavior at defect lines in the *dilute* Ising model. We consider strongly diluted systems, so that the bulk critical region is clearly controlled by the random fixed point, and insert the line defects as local perturbation. Then the relevance-irrelevance criterion for the local critical behavior is expected to have the same form as described above, with the exponents, ν and $\gamma_{1,1}$, referring now to the dilute model. Determining the local magnetization exponent β_d , at the defect, one may imagine two scenarios: (i) β_d showing a continuous variation with the defect coupling J_d , or (ii) β_d staying constant in, at least, some extended range of J_d . In the first case, there would be evidence that the marginality conditions, see Eq. (4), remain valid for the dilute model. Otherwise, one might infer that the critical exponents ν and $\gamma_{1,1}$ for the pure and dilute models are different.

In what follows we consider a random-bond nearest neighbor Ising model on a square lattice where the random ferromagnetic couplings, J_1 and J_2 , occur with equal probability. That model is self-dual,⁽¹²⁾ and the self-duality point

$$\tanh(J_1/T_c) = \exp(-2J_2/T_c) \quad (5)$$

corresponds to the critical point, if there is one phase transition in the system. Indeed, this assumption is strongly supported by numerical calculations.⁽⁴⁾ In this dilute model, ladder and chain defects are then introduced, where the defect couplings J_d are uniform and ferromagnetic.

To calculate the local critical properties we did extensive Monte Carlo (MC) simulations using Wolff's cluster flip algorithm.⁽¹³⁾ We have considered square lattices with L columns and L rows; J_d couples neighboring spins in the center column for chain defects, whereas for the ladder defect J_d connects spins between the two center columns. Typically we took $L = 256$, applying full periodic boundary conditions and generating about 10^4 clusters per realization. The results are then averaged over hundred realizations. The statistical errors during a MC run in a given sample turned out to be significantly smaller than those arising from the ensemble averaging. We mention that similar parameters were used in the previous study on the surface critical behavior of the dilute Ising model,⁽¹⁴⁾ which corresponds to the case with a ladder defect, where J_d vanishes.

In the MC simulations we calculated the average magnetization per column, $m(i) = \langle |\sum s_{ij}| \rangle / L$, where the sum runs over $j = 1, 2, \dots, L$. The defect magnetization is then given by $m_d = m(L/2)$. The simulations were performed at three values of the dilution parameter $r = J_1/J_2 = 1, 1/4$ and $1/10$ and for several values of the defect coupling in the region $0 \leq J_d/J_2 \leq 4$.

The magnetization profile $m(i)$ displays at the defect either a maximum or a minimum depending on the strength of the defect couplings, as illustrated in Fig. 2 for ladder defects. Far from the defect, there is a plateau in the profile with the height signaling the bulk magnetization, m_b . The size of the defect region, l_d , where the magnetization differs substantially from its bulk value, is related to the bulk correlation length of the system.

In the thermodynamic limit, $L \rightarrow \infty$, as the critical temperature T_c , see Eq. (5), is approached, the magnetization profile $m(i)$ goes to zero as a power-law $m(i) \sim t^{\beta(i)}$, where $\beta(L/2) = \beta_d$ and $\beta(i) = \beta$ for $|L/2 - i| > l_d$, where β is the usual bulk critical exponent. To estimate the values of these critical exponents from simulational data, one may define temperature dependent effective exponents

$$\beta(i)_{\text{eff}} = d \ln[m(i)] / d \ln[t] \quad (6)$$

which are approximated by using data at discrete temperatures, say, $t + \Delta t/2$ and $t - \Delta t/2$. In the limit of sufficiently small Δt and t , the effective exponents approach the true critical exponents, presuming that the system is large enough so that finite-size effects play no role. To avoid finite-size effects, L should be much larger than the correlation lengths in the bulk

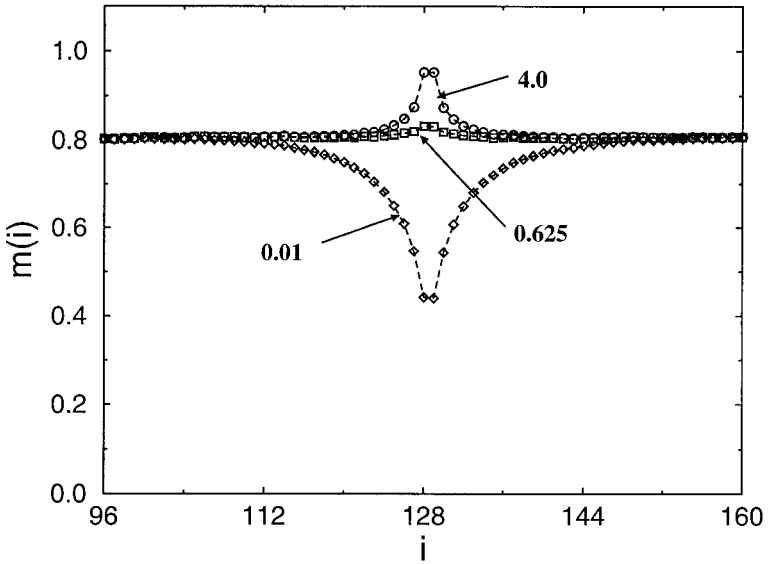


Fig. 2. Magnetization profiles in the dilute Ising model with a ladder defect at different couplings $J_d/J_2 = 4.0, 0.625$ and 0.01 at the dilution $r = J_1/J_2 = 1/4$ and reduced critical temperature $t = 0.06$. MC systems of size 256×256 were simulated. Only the central part is shown.

and at the defect line. Actually we approached the critical point by calculating $\beta(i)_{\text{eff}}$ for $t = 0.15, 0.13, 0.11, 0.09$ and 0.07 , with $\Delta t = 0.02$, and then did a linear extrapolation to $t = 0$. The error caused by the extrapolation seems to be rather small. Further technical details can be found in ref. 14.

Before presenting our findings about the defect line problem in the dilute model, we will first consider the perfect model with random defect couplings. The aim of this part of our investigation is to clarify, whether random defects could lead to varying local exponents. The two random, ferromagnetic couplings in the defect line, J and J_d , are assumed to occur with equal probability, where J is also the coupling in the rest of the system. Results about the local magnetization exponent β_d for various values of the ratio J_d/J are shown in Fig. 3, both for chain and ladder defects. The error bars in Fig. 3 take into account the sample averaging and the extrapolation. Only a few typical error bars are shown.

For $J_d/J = 1$, the critical exponent of the perfect model, $\beta = 1/8$, is reproduced quite accurately. For other values of that ratio, one observes a non-universal critical behavior with β_d varying continuously with J_d/J , in accordance with the above marginality conditions. It may be interesting to note the touching of the curves for the ladder and the chain defects at $J_d = J$.

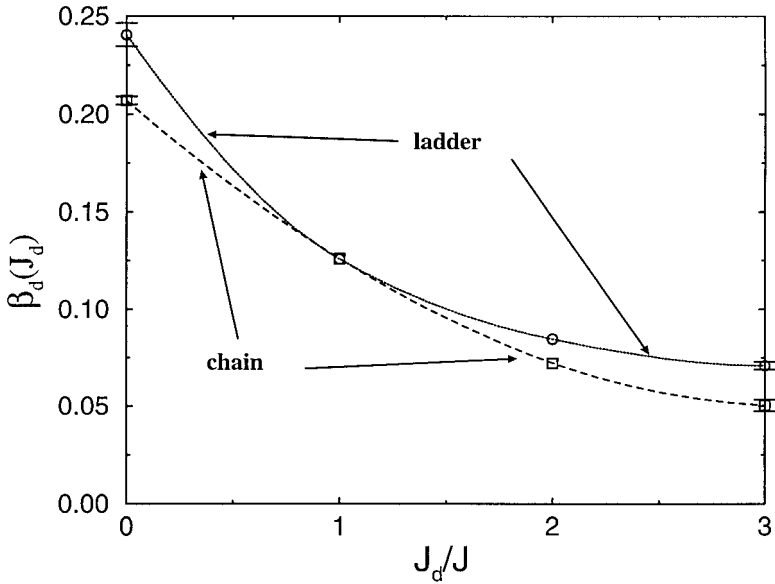


Fig. 3. Local magnetization exponent, β_d , of the pure Ising model with randomly distributed couplings in the chain and ladder type defects, see text. The lines are guides to the eye.

In Fig. 4, results about the critical exponent of the local magnetization β_d for the dilute model, with random couplings J_1 and J_2 , and the defect line, with uniform coupling J_d , are depicted. In the case $r = J_1/J_2 = 1$, our data are in good agreement with the exact results, see Eqs. (2) and (3) for chain and ladder defects. In the dilute case, $J_1 \neq J_2$, β_d is seen to vary continuously with the strength of the defect coupling J_d . For a fixed value of J_d , the defect energy-density increases, relative to the average bulk value, for decreasing value of $r = J_1/J_2$. Therefore, generally, there is an increasing local order at the defect, which is connected to a decreasing value of the defect exponent, β_d . This argument, however, seems to be not valid for the ladder defect with $J_d \gg J_2$. In this limit one has effectively a chain defect with random couplings $J_1 + J_2$, with probability 1/2, as well as $2J_1$ and $2J_2$, each with probability 1/4. Then, shown in Fig. 4, $\beta_d(J_d)$ is increasing with increasing dilution.- For $J_d = 0$, one recovers^(14, 15) the surface critical exponent $\beta_d = \beta_1 = 1/2$.

Another limiting situation is obtained for a chain defect with zero defect bond, $J_d = 0$. Then the problem is equivalent to a ladder defect with three random couplings, which depend on J_1 and J_2 . As seen in the inset

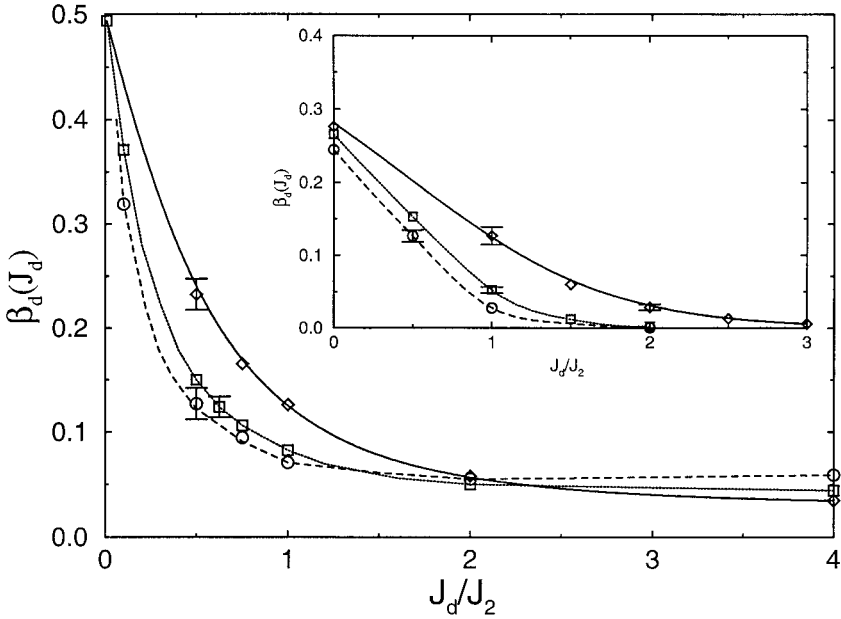


Fig. 4. Local magnetization exponent, β_d , of the random Ising model at different dilutions ($r = J_1/J_2 = 1$ diamond, $r = 1/4$ square, $r = 1/10$ circle) for ladder defects. Findings on the chain defect are shown in the inset. The exact results for the pure system with $r = 1$ are denoted by full line; the broken and dotted lines are guides to the eye.

of Fig. 4, such random couplings could also lead to a non-universal behavior. This observation is in agreement with our findings on the perfect model in Fig. 3.

To summarize, we considered uniform ladder and chain defects in two-dimensional dilute Ising models and determined the critical exponent of the defect magnetization. The exponent was found to be a continuous function of the defect coupling. Assuming that the first stability criterion mentioned above holds for the dilute case as well, one gets for the critical exponent ν of the bulk random Ising model the borderline value $\nu = 1$. Accordingly, one could rule out $\nu > 1$, as had been suggested before⁽⁷⁾ in the context of dilution dependent bulk critical exponents.

In conclusion, we suggest that the non-universal critical behavior is related to the borderline values of the critical exponents of the bulk dilute model, as given in Eq. (4). Consequently, one obtains $\nu = 1$ and $\gamma_{1,1} = 0$ (implying $\beta_1 = 1/2$, in agreement with ref. 14), both for the perfect and dilute two dimensional Ising models.

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